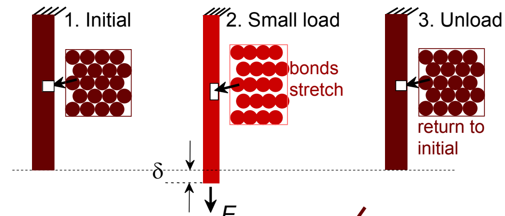
Mechanical Properties

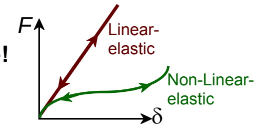
ISSUES TO ADDRESS...

- Stress and strain: What are they and why are they used instead of load and deformation?
- Elastic behavior: When loads are small, how much deformation occurs? What materials deform least?
- Plastic behavior: At what point does permanent deformation occur? What materials are most resistant to permanent deformation?
- Toughness and ductility: What are they and how do we measure them?

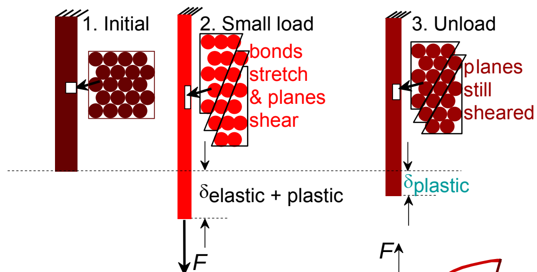
Elastic Deformation



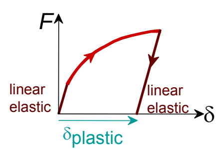
Elastic means reversible!



Plastic Deformation (Metals)

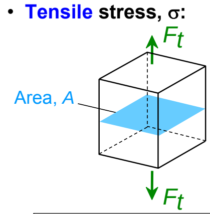


Plastic means permanent!



Engineering Stress

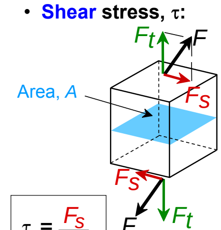
- **Tensile stress, σ:**



$$\sigma = \frac{F_t}{A_o} = \frac{lb_f}{in^2} \text{ or } \frac{N}{m^2}$$

original area before loading

- **Shear stress, τ:**

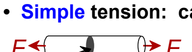


$$\tau = \frac{F_s}{A_o}$$

∴ Stress has units: N/m^2
i.e., same as Pressure: Pascals (Pa)
1 atm = 101,325 Pa = 14.7 psi

Common States of Stress

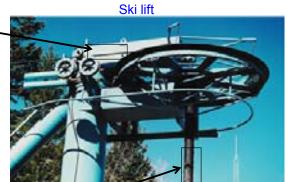
- **Simple tension: cable**



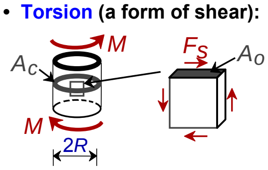
A_o = cross sectional area (when unloaded)

$$\sigma = \frac{F}{A_o}$$

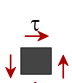
Ski lift



- **Torsion (a form of shear): drive shaft**



$$\tau = \frac{F_s}{A_o}$$



Note: $\tau = M/A_cR$ here.

OTHER COMMON STRESS STATES

- **Simple compression:**

Balanced Rock, Arches National Park
(photo courtesy P.M. Anderson)

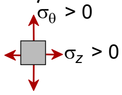
Canyon Bridge, Los Alamos, NM
(photo courtesy P.M. Anderson)

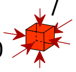
$$\sigma = \frac{F}{A_o}$$

Note: compressive structure member ($\sigma < 0$ here).

OTHER COMMON STRESS STATES

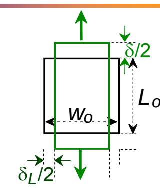
- **Bi-axial tension:**
- **Hydrostatic compression:**



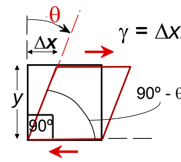


Strain

- **Tensile strain:** $\varepsilon = \dfrac{\delta}{L_o}$
- **Lateral strain:** $\varepsilon_L = \dfrac{-\delta_L}{w_o}$



- **Shear strain:**

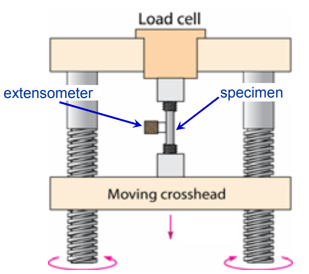


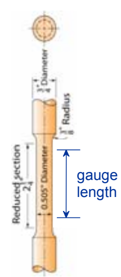
$\gamma = \Delta x/y = \tan\theta$

Strain is always dimensionless.

Stress-Strain Testing

- **Typical tensile test machine**
- **Typical tensile specimen**





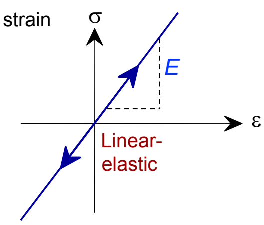
Adapted from Fig. 6.3, *Callister 7e.* (Fig. 6.3 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

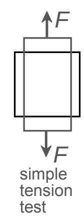
Linear Elastic Properties

- **Modulus of Elasticity:** *E* **(also known as Young's modulus)**
- **Hooke's Law:**

$\sigma = E\,\varepsilon$

stress = *E* ∗ strain

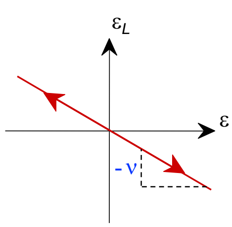




Poisson's ratio, ν

- **Poisson's ratio, ν:**

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$



metals: ν ~ 0.33
ceramics: ν ~ 0.25
polymers: ν ~ 0.40

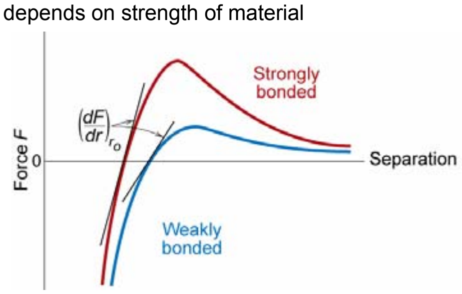
Units:
E: [GPa] ~ 10^5 atm
ν: dimensionless

ν > 0.50 density increases

ν < 0.50 density decreases (voids form)

Mechanical Properties

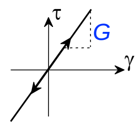
- Slope of stress-strain plot proportional to the elastic modulus.
- depends on strength of material

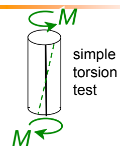


Other Elastic Properties

- **Elastic Shear modulus, G:**

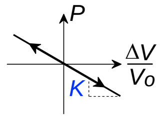
$$\tau = G\,\gamma$$

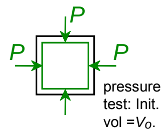




- **Elastic Bulk modulus, K:**

$$P = -K\frac{\Delta V}{V_o}$$



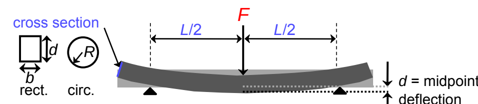


- **Special relations for isotropic materials:**

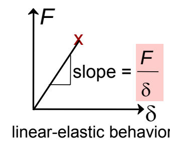
$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Measuring Elastic Modulus

- Room T behavior is usually elastic, with brittle failure.
- 3-Point Bend Testing often used.
  tensile tests are difficult for brittle materials.



- Determine elastic modulus according to:

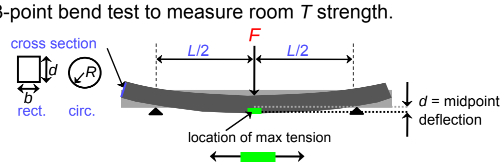


$$E = \frac{F}{\delta}\frac{L^3}{4bd^3} = \frac{F}{\delta}\frac{L^3}{12\pi R^4}$$

rect. cross section — circ. cross section

Measuring Strength

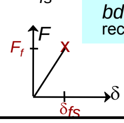
- 3-point bend test to measure room T strength.



- Flexural strength:

$$\sigma_{fs} = \frac{1.5F_f L}{bd^2} = \frac{F_f L}{\pi R^3}$$

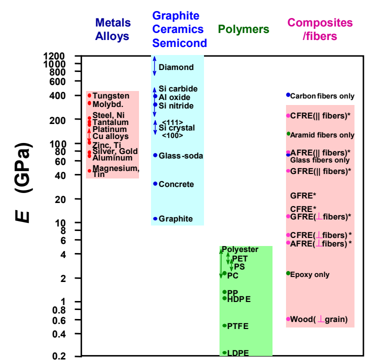
rect.



- Typical values:

Material	σ_{fs}(MPa)	E(GPa)
Si nitride	250-1000	304
Si carbide	100-820	345
Al oxide	275-700	393
glass (soda)	69	69

Comparison of Young's Moduli (tension)



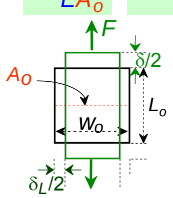
GPa = 10^9 Pa
= 10,000 atm

Based on data in Table B2, *Callister 7e.*
Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

Useful Linear Elastic Relationships

- **Simple tension:**

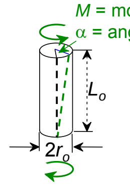
$$\delta = \frac{FL_o}{EA_o} \qquad \delta_L = -\nu\frac{Fw_o}{EA_o}$$



- **Simple torsion:**

$$\alpha = \frac{2ML_o}{\pi r_o^4 G}$$

M = moment
α = angle of twist

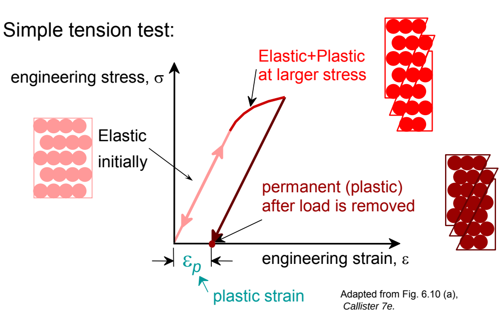


- Material, geometric, and loading parameters all important.

Plastic (Permanent) Deformation

(at lower temperatures, i.e. $T < T_{melt}/3$)

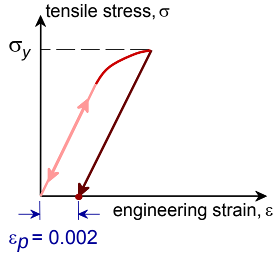
- Simple tension test:



Adapted from Fig. 6.10 (a), *Callister 7e.*

Yield Strength, σ_y

Yield Strength = YS = σ_y = Stress at which *noticeable* plastic deformation occurs (when ε_p ~ 0.002).

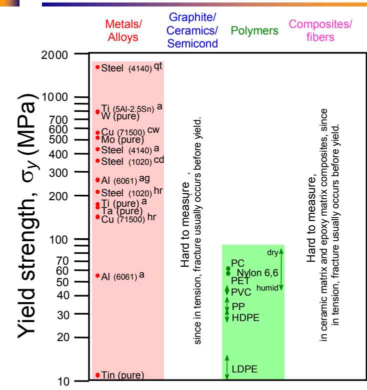


Note: for 2 inch sample

$\varepsilon = 0.002 = \Delta z/z$

$\therefore\ \Delta z = 0.004$ in

Yield Strength : Comparison

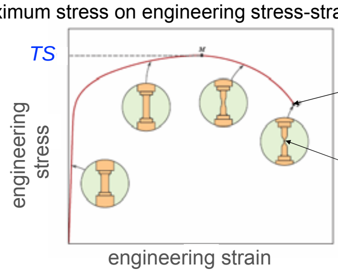


Room *T* values

Based on data in Table B4, *Callister 7e.*
a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

Tensile Strength, TS

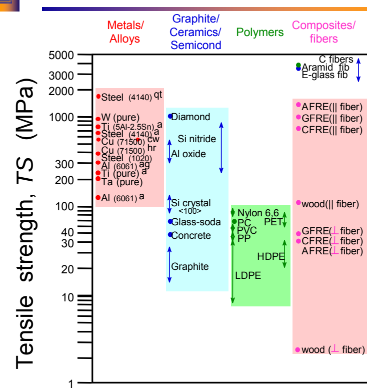
- Maximum stress on engineering stress-strain curve.



Adapted from Fig. 6.11, *Callister 7e.*

- Metals: occurs when noticeable necking starts.
- Polymers: occurs when polymer backbone chains are aligned and about to break.

Tensile Strength : Comparison

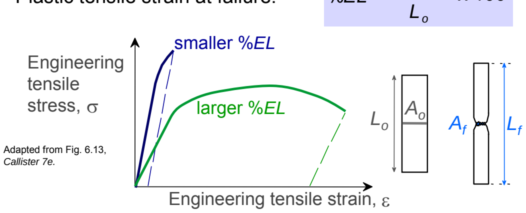


Room Temp. values

Based on data in Table B4, *Callister 7e.*
a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered
AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

Ductility

- Plastic tensile strain at failure: $\%EL = \frac{L_f - L_o}{L_o} \times 100$

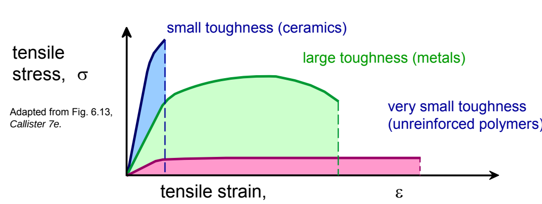


Adapted from Fig. 6.13, *Callister 7e.*

- Another ductility measure: $\%RA = \frac{A_o - A_f}{A_o} \times 100$

Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.



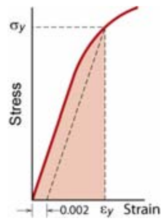
Adapted from Fig. 6.13, *Callister 7e.*

Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

Resilience, U_r

- Ability of a material to store energy

  Energy stored best in elastic region



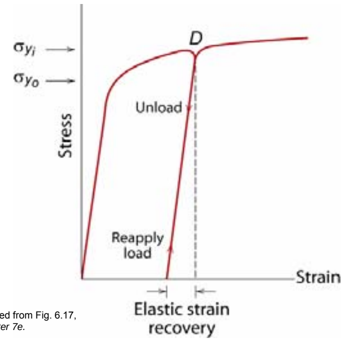
Adapted from Fig. 6.15, *Callister 7e.*

$$U_r = \int_0^{\varepsilon_y} \sigma \, d\varepsilon$$

If we assume a linear stress-strain curve this simplifies to

$$U_r \cong \frac{1}{2}\sigma_y \varepsilon_y$$

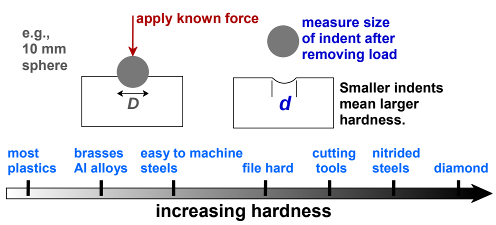
Elastic Strain Recovery



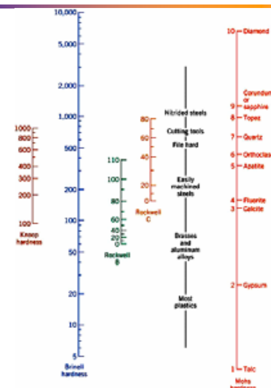
Adapted from Fig. 6.17, *Callister 7e.*

Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  resistance to plastic deformation or cracking in compression.
  better wear properties.



Hardness Scale: Comparison



Hardness: Measurement

- **Rockwell**

  No major sample damage

  Each scale runs to 130 but only useful in range 20 -100.

  Minor load 10 kg

  Major load 60 (A), 100 (B) & 150 (C) kg

  A = diamond, B = 1/16 in. ball, C = diamond

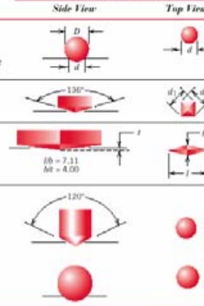
- **HB = Brinell Hardness**

  TS (psia) = 500 x HB

  TS (MPa) = 3.45 x HB

Hardness: Measurement

Table 6.4 Hardness Testing Techniques

Test	Indenter	Shape of Indentation: Side View	Shape of Indentation: Top View	Load	Formula for Hardness Number[a]
Brinell	10-mm sphere of steel or tungsten carbide			P	$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid	$l/b = 7.11$, $b/t = 4.00$		P	$HK = 14.2P/l^2$
Rockwell and Superficial Rockwell	Diamond cone; $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres			60 kg, 100 kg, 150 kg (Rockwell); 15 kg, 30 kg, 45 kg (Superficial Rockwell)	

[a] For the hardness formulas given, P (the applied load) is in kg, while D, d, d_1, and l are all in mm.

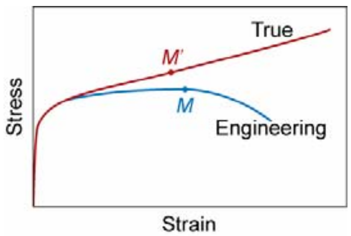
Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

True Stress & Strain

Note: Surf. Area changes when sample stretched

- True stress $\sigma_T = F/A_i$
- True Strain $\varepsilon_T = \ln(\ell_i/\ell_o)$

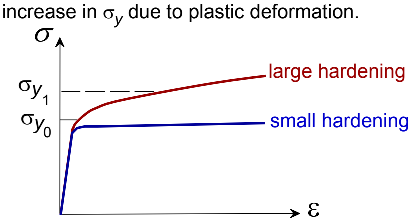
$$\sigma_T = \sigma(1+\varepsilon)$$
$$\varepsilon_T = \ln(1+\varepsilon)$$



Adapted from Fig. 6.16, *Callister 7e.*

Hardening

- An increase in σ_y due to plastic deformation.



- Curve fit to the stress-strain response:

$$\sigma_T = K(\varepsilon_T)^n$$

"true" stress (F/A)

"true" strain: $\ln(L/L_o)$

hardening exponent: n = 0.15 (some steels) to n = 0.5 (some coppers)

Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, N

$$\sigma_{working} = \frac{\sigma_y}{N}$$

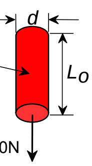
Often N is between 1.2 and 4, i.e., "Rule of Pi"

- Example: Calculate a diameter, d, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

$$\sigma_{working} = \frac{\sigma_y}{N}$$

$$\frac{220{,}000N}{\pi(d^2/4)} \qquad 5$$

1045 plain carbon steel:
σ_y = 310 MPa
TS = 565 MPa



F = 220,000N

d = 0.067 m = 6.7 cm

Summary

- Stress and strain: These are size-independent measures of load and displacement, respectively.
- Elastic behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (E or G).
- Plastic behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches σ_y.
- Toughness: The energy needed to break a unit volume of material.
- Ductility: The plastic strain at failure.