Problem 2

Time Evolution of Two-Level-Systems

Consider again the TLS Hamiltonian of problem 1,
\[ \hat{H} = \hbar \Omega (|R\rangle\langle L| + |L\rangle\langle R|) + \varepsilon |L\rangle\langle L|. \]

Suppose at \( t = 0 \) the TLS is prepared in the state
\[ |\Psi(0)\rangle = c_R(0)|R\rangle + c_L(0)|L\rangle. \]

At later times, the evolution of this state,
\[ |\Psi(t)\rangle = c_R(t)|R\rangle + c_L(t)|L\rangle \]
is characterized by the coefficients \( c_R(t), c_L(t) \).

(a) Substitute in the time-dependent Schrödinger equation and follow the general procedure described in class to obtain differential equations for the expansion coefficients.

(b) First focus on the symmetric case, \( \varepsilon = 0 \). Using Mathematica (or any other similar program), solve these differential equations for an initially right-localized state. Calculate the survival probability, \( P(t) = |\langle \Psi(t)|\Psi(0)\rangle|^2 \), and plot it.

(c) A position operator can be defined as
\[ \hat{x}_{\text{TLS}} = |R\rangle\langle R| - |L\rangle\langle L|. \]

Obtain an expression for the expectation value of position as a function of time in terms of the time-dependent coefficients. Plot the average position.

(d) Re-examine the evolution of the survival probability and average position for asymmetric two-level systems with \( \varepsilon = 1 \) and \( \varepsilon = 5 \). What do you observe?